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TECHNICAL NOTES

A step change in wall heat flux in a turbulent channel flow

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INTRODUCTION

THE DIFFUSION of temperature downstream of a step change in wall temperature or wall heat flux in a turbulent shear flow is of interest from theoretical and practical points of view. It is well known that, due to the quasi-linearity of the mean enthalpy equation, the solution to the stepwise discontinuity in wall temperature can be used to calculate an arbitrary wall temperature distribution [1]. Measurements, which have been made mainly in a turbulent boundary layer [2–4], have been useful for testing various calculation methods [5].

To provide experimental data comparable to those previously obtained in a boundary layer, we consider here a step change in wall heat flux introduced at one wall of a fully developed channel flow (see Fig. 1). The other (opposite) wall is maintained at approximately ambient temperature. The results are compared with those for a thermally fully developed flow obtained in a previous investigation [6] in which the full length of the channel wall was heated at a constant temperature. Hereafter, the fully developed thermal flow will be referred to as I; experimental details for I can be found in ref. [6].

EXPERIMENTAL CONDITIONS

Measurements were made in a fully developed turbulent channel flow at a Reynolds number *Re* equal to 3300. The channel aspect ratio of 18 and the measurement locations ($x/\delta > 250$, where x is measured from the working section entrance; the channel half-width δ is 21 mm) were sufficiently large to ensure a two-dimensional fully developed mean flow [7]. The channel walls were made of aluminium and perspex, respectively. The aluminium wall (1.27 cm thick) consisted of four plates, each of which could be heated separately by Sierracin pads (0.1 mm thick) connected in series and arranged in two rows of six along the length of each plate. The pads are bonded to the backs of the plates, thermal insulation (45 mm thick) ensuring that the heat loss was small. Only one (the most downstream) of the four plates was heated. The amount of heat supplied was controlled and the temperature of that plate continuously monitored using integrated-circuit temperature transducers embedded in small holes (using a highly conductive silicone compound) drilled at three x locations in the back of the plate. The temperature of the plate was homogeneous in the spanwise direction. The perspex wall was sufficiently thick (19 mm) to represent a reasonable approximation to a constant temperature boundary condition. The difference $T_w - T_\infty$ was maintained at about 10°C (sufficiently small for temperature to be considered a passive scalar). The temperature of the perspex wall was approximately T_∞. The heating origin was at $x_1 = 256\delta$ and measurements were made at six downstream locations ($\xi/\delta = 4.1, 7.6, 17.1, 21.4, 32.4, 47.6$). The friction velocity U_τ was obtained by the Preston tube method. Its value (0.133 m s^{-1}) agreed ($\pm 2\%$) with the value estimated from the streamwise pressure gradient.

Measurements were made with a single cold wire (1.2 mm long, 1.27 μm dia. Pt–10% Rh) which was operated in a constant current (50 μA) circuit and traversed across the channel with a mechanism with a least count of 0.01 mm. The initial distance of the wire from the wall was determined using the reflection method and a theodolite. A d.c. offset voltage was applied to the signal from the constant current circuit before it was amplified and low-pass filtered at a cut-off frequency of 1.75 kHz. The signal was then digitized using a personal computer (12 bit A/D converter, the sampling frequency was 3.5 kHz) and the data subsequently transferred (via an ETHERNET optical cable link) to a VAX 780 computer for analysis.

RESULTS

The mean temperature $\bar{T}^+ = (T_w - \bar{T})/T_\tau$ was measured across the thermal layer at six different positions downstream of the starting point of heating. The results are shown in

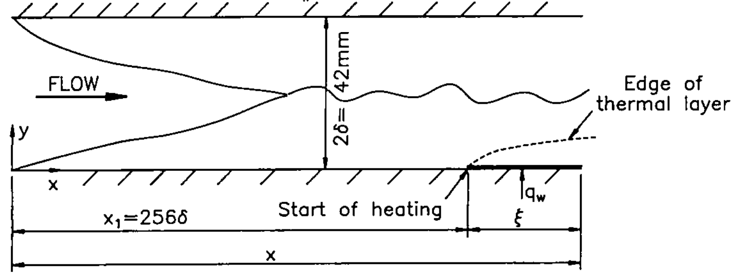

FIG. 1. Schematic arrangement of fully developed channel flow with a step change in heat flux applied to one of the walls.

NOMENCLATURE

c_f	skin friction coefficient, $\tau_w/\frac{1}{2}U_0^2$
c_p	specific heat at constant pressure
h	heat transfer coefficient
k	thermal conductivity
Nu	Nusselt number, $2\delta h/k = 2St\,Pr\,Re$
Pr	molecular Prandtl number, ν/α
Pr_T	turbulent Prandtl number, ν_T/α_T
q_w	thermometric wall heat flux
Re	Reynolds number, $U_0\delta/\nu$
St	Stanton number, $q_w/U_0(T_w-T_0)$
$\bar{T}$	mean temperature
T_τ	friction temperature, q_w/U_τ
T_w	wall temperature
T_∞	ambient temperature
T_0	mean temperature at the centreline
U_0	centreline velocity
U_∞	free stream velocity
U_τ	friction velocity, $\tau_w^{1/2}$
x, y	coordinates: x, streamwise; y, normal to the wall
x_1	unheated starting length.

Greek symbols

α	thermal diffusivity, $k/\rho c_p$
α_T	turbulent thermal diffusivity
δ	channel half-width
δ^+	Reynolds number, $\delta U_\tau/\nu$
Δ	boundary layer thickness
Δ_H	thermal layer thickness
θ	temperature fluctuation
ν	kinematic viscosity
ξ	distance downstream of heating origin, $x-x_1$
ρ	density
τ_w	kinematic wall shear stress.

Subscripts

w	wall value
0	centreline value
∞	ambient temperature.

Superscripts

$'$	r.m.s. value
$+$	normalization by wall variables U_τ, T_τ, ν
$-$	conventional time average.

Fig. 2 together with the distribution for I. To within the experimental scatter, there is reasonable collapse for $\bar{T}^+$ for the region $y^+ = yU_\tau/\nu \lesssim 30$ and all values of ξ/δ. The distribution in this region is also in good agreement with the mean temperature distribution for I. The good collapse for $\bar{T}^+$ in the wall region is partially 'forced' since T_τ was inferred from $(\partial\bar{T}/\partial y)_{y=0}$. This latter gradient was estimated (to an accuracy of $\pm 5\%$) from the slope of the mean temperature profile in the region $y^+ \lesssim 6$. In the region $y^+ \gtrsim 30$, there is an apparent increase in $\bar{T}^+$, for a fixed y^+, as ξ/δ increases, with an asymptotic approach to I. The difference, relative to I, in $\bar{T}^+$, is largest near the centreline, reflecting the streamwise growth of the thermal layer.

In contrast to the mean temperature, the r.m.s. temperature (Fig. 3) appears to require a relatively large distance before wall scaling is established, at least for the wall region. This trend, in qualitative agreement with that in the boundary layer [3], would imply a relatively slow streamwise evolution of the terms in the temperature variance budget (normalized by wall variables). At large values of ξ/δ, the largest differences in θ'^+, relative to I, occur near the centre of the channel, as in Fig. 2. For the present experimental conditions, the length of the cold wire is about six times the Kolmogorov length scale in the near-wall region. One would therefore expect θ' to be slightly underestimated due to the attenuation of the high wavenumber part of the temperature spectrum. It is difficult to estimate this error since the use of shorter wires is precluded by the need to keep the wire length to diameter ratio sufficiently large to avoid end conduction effects (e.g. ref. [8]).

The Stanton number St is shown in Fig. 4 together with the ratio $2St/c_f$ (the Reynolds analogy factor) and the Nusselt number $Nu = 2\delta h/k = 2St\,Pr\,Re$. St decreases with x and asymptotes to the value ($\simeq 3.3\times10^{-3}$) for I. Similar behaviours are observed for the Reynolds analogy factor which asymptotes to 1.14 and for the Nusselt number which asymptotes to 15.4.

An empirical expression for St was obtained [9] for a boundary layer ($Pr = 1$) with an unheated starting length x_1

$$St = \frac{c_f}{2}\left[1-\left(\frac{1}{1+(\xi/x_1)}\right)^{9/10}\right]^{-1/9}. \quad (1)$$

This relation (with $x_1 = 256\delta$ and $c_f = 6\times10^{-3}$), is in quite close agreement with the measured values of St, the Reynolds analogy factor and the Nusselt number (Fig. 4).

Several simplifying assumptions, e.g. ref. [9], were used in the derivation of (1), which was obtained by solving the integral energy equation. One-seventh power laws were assumed for (U/U_τ) and $(T_w-\bar{T})/(T_w-T_\infty)$, the molecular and turbulent Prandtl numbers were assumed to be equal to one, while Δ, the boundary layer thickness, was assumed to vary as $x^{4/5}$. This latter assumption is clearly not valid in the present case (δ = constant); also, the rate of growth for Δ_H ($\sim\xi^{0.35}$), estimated from the present experiments, is smaller than that ($\sim\xi^{0.70}$) in the boundary layer [2]. It seems unlikely

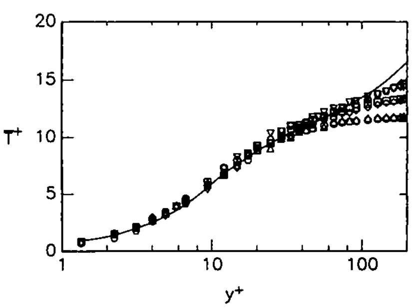

FIG. 2. Mean temperature distributions and comparison with a thermally fully developed flow (I). ○, $\xi/\delta = 4.1$; △, 7.6; □, 17.1; ◇, 21.4; ▽, 32.4; ⬡, 47.6. I: ——, $x/\delta = 279$.

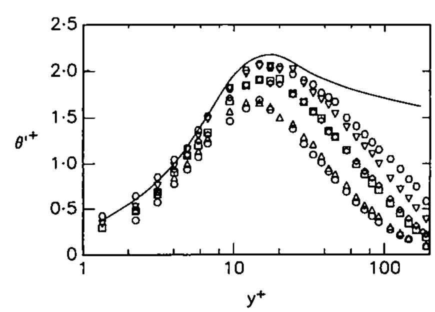

FIG. 3. R.m.s. temperature distributions and comparison with thermally fully developed flow (I). Symbols are as in Fig. 2.

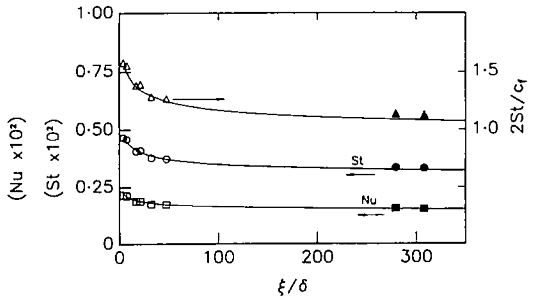

FIG. 4. Stanton number St, Nusselt number Nu and Reynolds analogy factor $2St/c_f$. Closed symbols refer to a thermally fully developed flow (I). ○, experimental values of St; △, $2St/c_f$; □, Nu; ——, equation (1).

however that this difference would have a significant effect on (1), which has been validated in the boundary layer when $\Delta_H \ll \Delta$ [9]. Assuming that the present value of Pr ($\simeq 0.72$) is sufficiently close to $Pr = 1$, the close agreement between (1) and the present values of St (Fig. 4) indirectly suggests that the assumption $Pr_T = 1$ (or Reynolds' analogy) should be reasonable, at least in the near-wall region, for the present situation. Direct numerical simulation data in the near-wall region of a thermally fully developed channel flow [10] have confirmed the validity of $Pr_T = 1$ (when Pr is near 1). It would certainly be of interest to extend this conformation to a developing thermal layer.

CONCLUSIONS

A step change in heat flux has been applied to one of the walls of a fully developed turbulent channel flow, while the other (opposite) wall is at approximately ambient temperature. In the near-wall region, scaling on wall variables is satisfied to a good approximation by the mean temperature but not by the r.m.s. temperature. Sufficiently downstream of the step, mean and r.m.s. temperature distributions asymptote to values obtained for a thermally fully developed flow. The streamwise variation of the Stanton number, Reynolds analogy factor and Nusselt number downstream of the heating origin is well described by an empirical relation obtained for a boundary layer with an unheated starting length.

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Transfer function method for analysis of temperature field

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INTRODUCTION

IN RECENT years a number of articles have been written on the wave model of heat propagation where the finite velocity of the heat wave is taken into account [1–7]. The wave model of heat propagation leads to a precise analysis of many physical phenomena which, when analysed by Fourier's law, will result in some errors [2].

Classical examples of the correctness of the heat wave model are intensive heating of solids by means of laser wave impulses of high amplitude and short duration [8], electromagnetic radiation [9], fast heat flow in rarefied media [10], etc.

Fourier's law, when used in the classical analysis of thermal problems, defines the dependence between heat flux intensity and time–space distribution of temperature T. By combining Fourier's law with the principle of energy conservation we obtain a parabolic equation of heat diffusion

$$\alpha \nabla^2 T = \frac{\partial T}{\partial t} \quad (1)$$

where $\alpha = k/\rho c$ is the diffusion coefficient and k, ρ and c are thermal conductivity, mass density and specific heat, respectively. A physical interpretation of the solution of equation (1) shows that the speed of heat propagation is infinite. In some of the cases mentioned above, there is, by necessity, a generalization of the mathematical model represented by equation (1). To achieve this we use a model of heat wave damping. Then Fourier's law undergoes modification and, in combination with the principle of energy